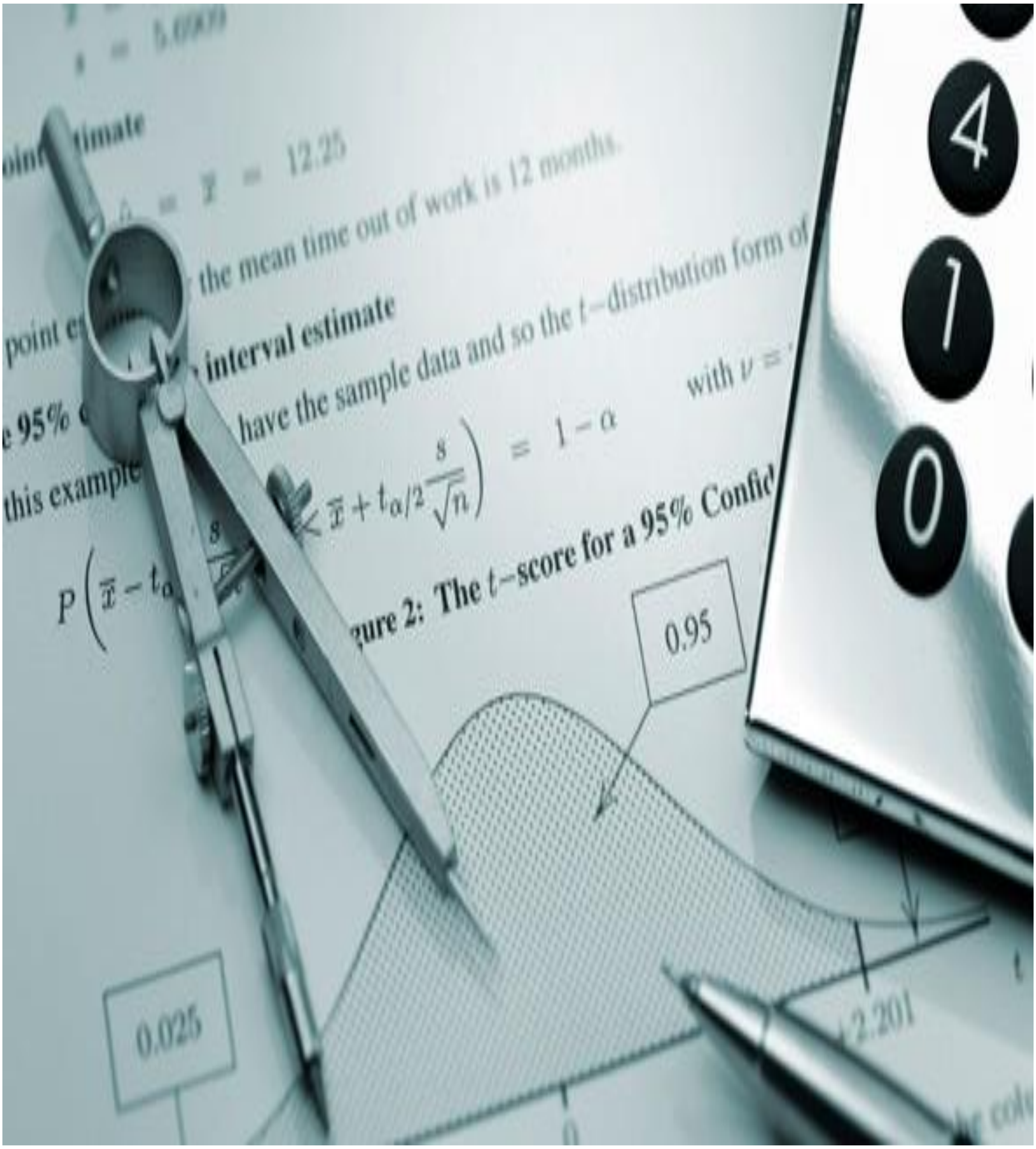
UNIVERSITY FOUNDATION PROGRAMME

MATHEMATICS SPECIFICATION

PREPARING STUDENTS FOR UNIVERSITY SUCCESS

FOR TEACHING FROM 2021



August 2021

CATS UFP

CATS UFP is a Level 3 course, specifically designed to help international students move successfully from secondary education to a UK University.

The CATS UFP is delivered over 420 directed hours of teaching and learning, over 3 subjects, and utilises a rigorous style of study, within a pastorally supportive and culturally stimulating environment that enables students' learning to develop and progress successfully. Students are able to access a variety of assessment methods that are common in UK Universities, such as portfolios, presentations academic posters, and examinations combined with content specifically designed to build on prior learning from courses around the world.

English for Academic purposes is an essential part of CATS UFP, and all students will take an English course that supports their learning and prepares them for university life, as well as having access to many extracurricular activities that further reinforce their use of English. Assessment design within each subject carefully focuses on subject knowledge and skills, rather than the ability to cope with English as a second language.

CATS Colleges provide a stimulating intellectual and diverse environment with small classes; thus, enabling the best learning to happen. With CATS UFP, all learning happens with teachers who have excellent subject knowledge and are expert in creating a positive learning environment for students from a wide range of backgrounds.

CATS UFP has a successful record of accomplishment and is highly respected by UK universities. With this qualification, students with 12 years of schooling from their own country can make the progression that they want, to a wide range of UK universities, including those ranked most highly for both research and teaching. CATS UFP has strong advocates in its alumni, who display what a CATS UFP qualification can give them. Graduates report that they feel very well prepared for university study; often, better prepared than students from other Level 3 programmes. Universities have confirmed this, through testimonials and through extensive consultation with university based External Examiners it has gained excellent credibility with UK universities.

VERSION 3 AMENDMENTS

Description of change	Page number
CATS Colleges logo replaced with CATS UFP logo	All pages
First teaching year amended	Page 1
Final examinations amended to problem sheet structure	Pages 16-17,
Formula Book Added	Pages 55 – 62

VERSION 2 AMENDMENTS

The following amendments have been made from the previous version of this specification:

Description of change	Page number
Clarification on the use of binomial probabilities	Page 11
Updated job titles from "Faculty Leader" to "Chief Examiner"	Page 20
Added clarification to the Coursework marking criteria in Appendix A	Pages 23 – 27
Added information to the Controlled Assessment Details	Page 28
Added clarification to the Controlled Assessment marking criteria in Appendix B	Pages 30 - 32

VERSION NOTES

Original specification written by Rob Mathers
Version two amendments added by Rob Mathers
Version three amendments added by Rob Mathers

INTRODUCTION

Why Choose Mathematics UFP?

Dynamic and engaging content:

For 35 years CATS UFP has provided a high quality, successful qualification. Through consistent improvement using teacher and student feedback, classroom experience and by working closely with universities the Mathematics UFP course is designed to engage international students through topics and issues that are relevant across the globe.

Real life skills:

In addition to learning standard mathematical formulae and techniques, students will develop their critical thinking skills, logic and reasoning ability through the various problem solving elements of the course. These skills are highly desirable in higher education and valued by employers. The experience students gain from analysing and interpreting data during the coursework and controlled assessment course elements is also highly valued.

Assessment success:

Mathematics UFP involves three methods of assessment: a piece of coursework, a controlled assessment and two examination papers.

- The coursework and controlled assessment give students the opportunity to analyse data and present a written report on their findings. Students will be assessed not only on their ability to calculate values using their data, but also to analyse and interpret these calculations.

- Our examination papers cover the entirety of the pure mathematics course content. The only topics not assessed in the examination are the statistical analysis skills covered by the coursework and controlled assessment. The style of these questions will involve a mix of applying standard techniques/formulae, interpreting/explaining and problem solving. However, these questions will be structured in a way that they are accessible for students with a lower level of English.

***AIMS OF THE COURSE**

The UFP encourages students to:

- Understand mathematics and mathematical processes in a way that promotes confidence, fosters enjoyment and provides a strong foundation for progress to further study.
- Extend their range of mathematical skills and techniques.
- Understand coherence and progression in mathematics and how different areas of mathematics are connected.
- Apply mathematics in other fields of study and be aware of the relevance of mathematics to the world of work and to situations in society in general.
- Use their mathematical knowledge to make logical and reasoned decisions in solving problems both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly.
- Reason logically and recognise incorrect reasoning.
- Generalise mathematically.
- Construct mathematical proofs.
- Use their mathematical skills and techniques to solve challenging problems that require them to decide on the solution strategy.
- Recognise when mathematics can be used to analyse and solve a problem in context.
- Represent situations mathematically and understand the relationship between problems in context and mathematical models that may be applied to solve them.
- Draw diagrams and sketch graphs to help explore mathematical situations and interprets solutions.
- Make deductions and inferences and draw conclusions by using mathematical reasoning.
- Interpret solutions and communicate their interpretation effectively in the context of the problem.
- Read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding.
- Read and comprehend articles concerning applications of mathematics and communicate their understanding.
- Use technology such as calculators and computers effectively and recognise when their use may be inappropriate.
- Take increasing responsibility for their own learning and the evaluation of their own mathematical development.

KEY SKILLS

Students taking this course will be encouraged to develop into independents learners with the ability to think critically, understanding the key importance of research and presentational skills. The course covers these key skills in the following ways:

Reasoning and critical thinking:

- Use problem solving skills to solve problems effectively in situations where more than one approach is possible.
- Select, organise and communicate relevant information in a variety of forms;
- Use mathematical techniques in a multitude of situations applicable to the real world

Independent Learning:

- Organise own learning through management of time and material;
- Work on own initiative to prioritise tasks;
- Work independently to support understanding of material;
- Carry out self-directed learning tasks.

Research Skills:

- Research an area of interest and find data suitable to analyse statistically
- Ensure all research is referenced and not plagiarised
- Use ICT to develop information literacy skills, to communicate and collaborate with others.

Presentational Skills:

- Systematic documentation of finding and analysis;
- Use of word processing and other forms for ICT for communication;
- Organise information clearly and coherently, using specialist vocabulary when appropriate

*ASSUMED PRIOR KNOWLEDGE

UK Government recommendations for Level 3 qualifications states that:

"…specifications must build on the skills, knowledge and understanding set out in the whole GCSE subject content for mathematics for first teaching from 2015."

We appreciate that UFP students come from a diverse range of cultures and backgrounds, so necessary level two content and terminology will be covered in UFP teaching.

It is assumed that students will be conversant with the following subject content before the start of the UFP Mathematics course. All GCSE content that overlaps with the UFP Mathematics Course content has been removed from the list below.

Number

- order positive and negative integers, decimals and fractions; use the symbols =, ≠, , ≤, ≥
- apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
- recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions; use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
- use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem
- apply systematic listing strategies including use of the product rule for counting
- use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5; estimate powers and roots of any given positive number
- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 or $\frac{3}{8}$); change recurring decimals into their corresponding fractions and vice versa
- identify and work with fractions in ratio problems
- interpret fractions and percentages as operators
- use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
- estimate answers; check calculations using approximation and estimation, including answers obtained using technology
- round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding

Algebra

- use and interpret algebraic notation, including:
    - ab in place of a × b
    - 3y in place of y + y + y and 3 × y
    - a^2 in place of a × a, a^3 in place of a × a × a, a^2b in place of a × a × b
    - $\frac{a}{b}$ in place of a ÷ b
    - coefficients written as fractions rather than as decimals
    - brackets
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, formulae, identities inequalities, terms and factors
- simplify and manipulate algebraic expressions (including those involving surds and algebraic fractions) by:
    - collecting like terms
    - multiplying a single term over a bracket
    - taking out common factors
    - expanding products of two or more binomials
- understand and use standard mathematical formulae; rearrange formulae to change the subject
- work with coordinates in all four quadrants
- solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph
- translate simple situations or procedures into algebraic expressions or formulae
- generate terms of a sequence from either a term-to-term or a position-to-term rule
- recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions

Ratio, proportion and rates of change

- change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
- use scale factors, scale diagrams and maps
- express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
- use ratio notation, including reduction to simplest form
- divide a given quantity into two parts in a given part:part or part:whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
- express a multiplicative relationship between two quantities as a ratio or a fraction
- understand and use proportion as equality of ratios
- relate ratios to fractions and to linear functions
- define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics
- solve problems involving direct and inverse proportion, including graphical and algebraic representations
- use compound units such as speed, rates of pay, unit pricing, density and pressure
- compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors
- understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; construct and interpret equations that describe direct and inverse proportion

Geometry and measures

- use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description
- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)
- apply angle facts and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
- use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)
- know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)
- know the formulae: circumference of a circle = 2πr = πd, area of a circle = πr 2 ; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes; surface area and volume of spheres, pyramids, cones and composite solids
- know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and the trigonometric ratios, sinθ = $\frac{opposite}{hypotenuse}$, cosθ = $\frac{adjacent}{hypotenuse}$ and tanθ = $\frac{opposite}{adjacent}$; apply them to find angles and lengths in right-angled triangles and, where possible, general triangles in two and three dimensional figures

Probability

- record describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees
- apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments
- relate relative expected frequencies to theoretical probability, using appropriate language and the 0 - 1 probability scale
- apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
- understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size

Statistics

- interpret and construct tables, charts and diagrams, including frequency tables

USE OF TECHNOLOGY

In this course students will be expected to work with both word processing and spreadsheet software, as well as using a graphical calculator. Appropriate calculators should include the following features:

- an iterative function
- the ability to compute summary statistics and access probabilities from standard statistical distributions.

USE OF DATA

Students are expected to be able to:

- use technology such as spreadsheets or specialist statistical packages to explore the data set(s)
- interpret real data presented in summary
- use data to investigate questions arising in real contexts

***SUBJECT CONTENT**

Examinable Content:

The topics stated in the table below will be assessed through the final examinations.

	Content
1. Algebra and Functions	• Understand and use the laws of indices for all rational exponents. • Use and manipulate surds, including rationalising the denominator. • Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the conditions for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown. • Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation. • Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions. • Represent linear and quadratic inequalities such as and graphically. Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem. Simplify rational expressions including by factorising and cancelling, and algebraic division (by linear expressions only). Understand and use graphs of functions; sketch curves defined by simple equations including polynomials, the modulus of a linear function, $y = \frac{a}{x}$ and $y = \frac{a}{x^2}$ (including their vertical and horizontal asymptotes); interpret algebraic solution of equations graphically; use intersection points of graphs to solve equations. Understand and use proportional relationships and their graphs. • Understand and use composite functions; inverse functions and their graphs • Understand the effect of simple transformations on the graph of $y = f(x)$, including sketching associated graphs: $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$. • Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear)
2. Coordinate geometry in the (x,y) plane	• Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; gradient conditions for two straight lines to be parallel or perpendicular. Be able to use straight line models in a variety of contexts. • Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$; completing the square to find the centre and radius of a circle; use of the following properties: • the angle in a semicircle is a right angle • the perpendicular from the centre to a chord bisects the chord the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point
3. Sequences and series	• Understand and use the binomial expansion of $(a + b)^n$ for positive integer *n*; the notations *n*! and *nCr;* link to binomial probabilities. • Work with sequences including those given by a formula for the *n*th term and those generated by a simple relation of the form x_{n+1}= f(x_n); increasing sequences; decreasing sequences; periodic sequences

	• Understand and use sigma notation for sums of series • Understand and work with arithmetic sequences and series, including the formulae for *n*th term and the sum to *n* terms • Understand and work with geometric sequences and series including the formulae for the *n*th term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$; modulus notation • Use sequences and series in modelling
4. Trigonometry	• Understand and use the definitions of sine, cosine and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form $\frac{1}{2}absinC$. • Work with radian measure, including use for arc length and area of sector • Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity • Know and use exact values of sin and cos for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples thereof, and exact values of tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \pi$ and multiples thereof. • Understand and use $tan\theta = \frac{sin\theta}{cos\theta}$ • Understand and use $sin^2\theta + cos^2\theta = 1$ • Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle. • Construct proofs involving trigonometric functions and identities • Use trigonometric functions to solve problems in context
5. Exponentials and logarithms	• Know and use the function a^x and its graph, where *a* is positive. • Know and use the function e^x and its graph. • Know that the gradient of is e^{kx} equal to ke^{kx} and hence understand why the exponential model is suitable in many applications. • Know and use the definition of $log_a x$ as the inverse of a^x, where *a* is positive and $x \geq 0$. • Know and use the functionand $ln\, x$ its graph. • Know and use as the inverse function of e^x. • Understand and use the laws of logarithms: $log_a x + log_a y = log_a x$y; $log_a x - log_a y = log_a \frac{x}{y}$; $klog_a x = log_a x^k$ including, for example, $k = -1$ and $k = \frac{-1}{2}$. • Solve equations of the form $a^x = b$. • Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for *x* and *y*. • Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models.
6. Differentiation	• Understand and use the derivative of as the gradient of the tangent to the graph of $y = f(x)$ at a general point (*x*, *y*); the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second derivatives. • Understand and use the second derivative as the rate of change of gradient. • Differentiate x^n, for rational values of *n,* and related constant multiples, sums and differences. • Differentiate e^x and a^{kx}, $sinkx$ and $coskx$ related sums, differences and constant multiples. • Understand and use the derivative of *ln*x.

	• Apply differentiation to find gradients, tangents and normals, maxima and minima and stationary points, points of inflection. • Identify where functions are increasing or decreasing. • Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions • Construct simple differential equations in pure mathematics and in context.
7. Integration	• Know and use the Fundamental Theorem of Calculus • Integrate x^n (excluding *n* = -1), and related sums, differences and constant multiples. • Integrate $e^{kx}, \frac{1}{x}, sinkx, coskx$ and related sums, differences and constant multiples. • Evaluate definite integrals; use a definite integral to find the area under a curve] and the area between two curves. • Carry out simple cases of integration by substitution and integration by parts. • Integrate using partial fractions that are linear in the denominator

COURSEWORK CONTENT:
The following topics will be assessed via Coursework:

8. Data presentation and interpretation	• Construct and interpret diagrams for discrete single-variable data • Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams • Understand informal interpretation of correlation • Understand that correlation does not imply causation • Be able to calculate measures of central tendency and variation, including standard deviation, for discrete data • Interpret measures of central tendency and variation, extending to standard deviation, for discrete data

CONTROLLED ASSESSMENT CONTENT:
The following topics will be assessed via the Controlled Assessment:

8. Data presentation and interpretation	• Construct and interpret diagrams for single-variable continuous data, including understanding that area in a histogram represents frequency • Be able to calculate measures of central tendency and variation, extending to standard deviation for continuous data • Interpret measures of central tendency and variation, extending to standard deviation for continuous data • Select or critique data presentation techniques and compare data in the context of a statistical problem: • Be able to calculate and compare the skew of two data sets

*ASSESSMENT OVERVIEW

Assessment Objectives

Assessment objectives (AOs) are designed for Level 3 Mathematics. CATS UFP places a strong emphasis on the use of maths in an international context when compared to other Level 3 qualifications

	Objective	Weighting
A01	**Use and apply standard techniques** Learners should be able to: • select and correctly carry out routine procedures; and • accurately recall facts, terminology and definitions	62%
A02	**Reason, interpret and communicate mathematically** Learners should be able to: • construct rigorous mathematical arguments (including proofs); • make deductions and inferences; • assess the validity of mathematical arguments; • explain their reasoning; and • use mathematical language and notation correctly. *Where questions/tasks targeting this assessment objective will also credit Learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'solve problems within mathematics and in other contexts' (AO3) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).*	16%
A03	**Solve problems within mathematics and in other contexts** Learners should be able to: • translate problems in mathematical and non-mathematical contexts into mathematical processes; • interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and imitations; • translate situations in context into mathematical models; • use mathematical models; and • evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them. *Where questions/tasks targeting this assessment objective will also credit Learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'reason, interpret and communicate mathematically' (AO2) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).*	22%

ASSESSMENT OBJECTIVE WEIGHTINGS PER COURSE ELEMENT

	Problem Sheets	Coursework	Controlled Assessment	**Overall**
Course weighting	80%	10%	10%	
A01 weighting	65%	50%	50%	**62%**
A02 weighting	10%	40%	40%	**16%**
A03 weighting	25%	10%	10%	**22%**

Assessment structure

Mathematics UFP involves three methods of assessment: a piece of coursework, a controlled assessment and three problem sheets. The controlled assessment gives students the opportunity to analyse data of their choice and present their findings in the form of a project. Students will be assessed on their ability to calculate values using their data and also to analyse and interpret these calculations. The problem sheets cover the entirety of the course content with the exception of the skills covered by the controlled assessment.

Problem Sheets:

Paper 1:	Paper 2:	Paper 3:
53 marks	53 marks	54 marks
1 hour 20 minutes	1 hour 20 minutes	1 hour 20 minutes
26.5% of the qualification	26.5% of the qualification	27% of the qualification

Examined Content:

- Algebra and Functions
- Coordinate geometry in the (x,y) plane
- Sequences and series
- Trigonometry
- Exponentials and logarithms
- Differentiation
- Integration

Assessment overview

- Students must answer all questions.
- Calculators may be used in this assessment.
- Problem Sheet 1 will assess content from:
    - Algebra and Functions
    - Coordinate Geometry
- Problem Sheet 2 will assess content from:
    - Sequences and Series
    - Trigonometry
- Problem Sheet 3 will assess content from:
    - Differentiation
    - Integration
    - Exponentials and Logarithms
- It is permitted for content from earlier problem sheets to appear in later papers, but not the other way around. For example, coordinate geometry may appear in paper 3, but integration may not appear in paper 1.

Note: the assessment objective weighting described in the table on the previous page will be taken as an average across all three Problem Sheets. For example, while AO1 will be approximately 65% across all three problem sheets, it is likely that it will be a slightly higher weighting in Problem Sheet 1 compared to Problem Sheets 2 and 3 due to the nature of the content being assessed here. Similarly, it is likely that AO2 and AO3 will be weighted more in Problem Sheets 2 and 3 for the same reason.

These examinations will take place under controlled conditions with the formula book from Appendix G.

Coursework:
20 marks

10% of the qualification

Examined content: data presentation and interpretation (discrete data)

Assessment overview

- Students will be given a week to research their data.
- After this time, marking criteria are administered. Students are given one week to produce the first draft of their coursework. Once this has been read through, staff will provide feedback on this first draft. This will be the only opportunity for students to get formal feedback on their work.
- Students will then be expected to work on their coursework independently and submit it before the deadline.
- Students should submit their final project as a Word file (or similar), in addition to the Excel file (or similar) that they have used to carry out all their calculations.
- Must be completed and submitted electronically.

Controlled Assessment:
20 marks

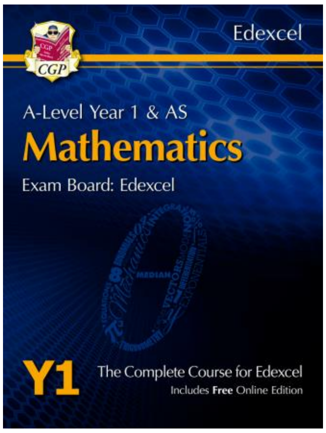
10% of the qualification

Examined content: Data presentation and interpretation (continuous data)

Assessment overview

- Administered in a two and a half hour session on a given date.
- Students will be given a set of continuous data to analyse. The assessment/marking criteria will be provided to them. They will be asked to work produce a synopsis of this data according to these marking criteria.
- They will be expected to perform calculations on the given data, to represent these data as a histogram and to compare and contrast the two data sets.
- At the end of this time, their work will be taken in and marked.

Suggested Reading



New A-Level Maths for Edexcel: Year 1 & AS Student Book with Online Edition

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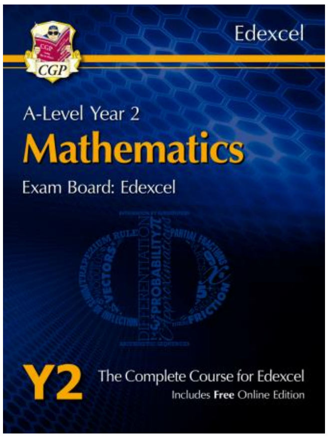

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1. Teaching Plan

Topic	No. of hours	Assessment Element
• Algebra and Functions	30	Examination
• Coordinate geometry in the (x,y) plane	10	Examination
• Sequences and series	15	Examination
• Trigonometry	15	Examination
• Exponentials and Logarithms	10	Examination
• Data presentation and interpretation (discrete data)	15	Coursework
• Data presentation and interpretation (continuous data)	10	Controlled Assessment
• Differentiation	20	Examination
• Integration	15	Examination
Total	140	

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APPENDIX A. COURSEWORK DETAILS

The Coursework is worth 10% of the qualification.

Students will be expected to analyse two different but related data sets of discrete data, which they have researched themselves, and interpret their findings. For example, students could investigate the relationship between population changes in two different countries and investigate the comparison between their respective growths/declines. In this investigation, they could be looking to conclude whether the population of country A will ever exceed the population of country B. Students will analyse measures of location and spread for each of the data sets individually, before calculating the regression and correlation of the two sets as bivariate data. For example, if students were comparing countries, they can find the average population over the time periods they are investigating, as well as variance and standard deviation, then they can calculate the PMCC and regression line between the two variables. Students can then draw a conclusion based on their findings. They can also evaluate the accuracy of their data and the results they have obtained, including references to any anomalous data values.

The marks will be awarded according to these criteria:

- 60% (12 marks) Mathematical Calculation
- 20% (4 marks) Data Interpretation
- 10% (2 marks) Conclusion
- 10% (2 marks) Structure and Communication (including Correct Terminology and Notation)

Students will be provided with the marking criteria for this project so they know what the markers are assessing. Initially, they will be given one week to research suitable data to analyse. At the end of this week, the students will be given the marking criteria and will have one week to complete their first draft. This is the only opportunity that students will have to recive formal feedback. Once this has been received, they will then be expected to work on this independently and submit it before the deadline. Students are expected to submit their project as a Word file, and their data in an Excel spreadsheet; the latter is used to by the marker to ensure that the calculations have been performed correctly.

Students will be encouraged to find their own data set through online research, however students will be directed to select websites and data sources if they are unable to find these on their own.

UFP MATHEMATICS COURSEWORK MARKING SHEET:

Name: ______________________________ Date: _________________

Mathematical Calculations (MC):	✓	×
MC1: Data is suitable to analyse (|PMCC|≥0.6)		
MC2: Attempt to find mean, median and mode (where appropriate) for both data sets		
MC3: Mean, median and mode (where appropriate) calculated correctly		
MC4: Attempt to find standard deviation and IQR for both data sets		
MC5: Standard deviation and IQR calculated correctly.		
MC6: Outliers searched for and identified (if any)		
MC7: Attempt to find Sxx, Syy, Sxy		
MC8: Sxx, Syy and Sxy calculated correctly		
MC9: PMCC calculated correctly		
MC10: Correct attempt made to find the equation of the regression line		
MC11: Values of a and b in regression equation calculated correctly with correct working shown.		
MC12: Correct scatter graph (appropriate scale and labelled axes) with regression line plotted accurately on scatter graph		
TOTAL:	**/12**	

Data Interpretation (DI):	✓	×
DI1: Correct comment on scatter graph (must address outliers, if any)		
DI2: Comparison of data as univariate		
DI3: Correct comment on PMCC (must refer to value), including idea that correlation does not imply causation		
DI4: Correct comment on the gradient (must refer to the value)		
TOTAL:	**/4**	

Conclusion (CN):	✓	×
CN1: Some correct ideas drawn from working towards answering the title question.		
CN2: Fully correct conclusion including comments on intricacies of the data. All ideas are justified from prior work.		
TOTAL:	**/2**	

Structure and Communication (SC):	✓	×
SC1: Attempt at logical structure including brief introduction.		
SC2: Full logical structure throughout. All ideas are appropriate and communicated clearly.* Complete correct notation throughout.		
TOTAL:	**/2**	

TOTAL		**/20**

* Note: Poor English should not be punished, but key words should be used and ideas should be clear.

All marks should be awarded from the student’s project written in Word. The Excel file may be used for ease of marking and to justify it is the student’s own work in the event of suspected plagiarism.

Marking Criteria clarification

*Note that **all** marks should be considered independent unless otherwise stated.*

Mathematical Calculations (MC):
MC1: Data is suitable to analyse ($|\text{PMCC}| \geq 0.6$):
In addition to the PMCC being greater than or equal to 0.6, the two data sets must each be two sets of data that could be analysed separately. There should be a minimum of 30 values in each data set for this mark to be awarded. It should not be one data set and another referential data set such as the time frame for it. For example, the two data sets could NOT be population of country A and the year: they would have to be two separate sets of population data for countries A and B. Data sets do not have to analyse the exact same type of variable (e.g. two sets of population data), but should represent two things that similar enough that an analysis is sensible.

- MC2: Attempt to find mean, median and mode (where appropriate) for both data sets:

This attempt should be evidenced from the calculation shown in the students' project (word file). Minimum amount needed for the mark to be awarded is the formula for the mean and median with values substituted in. Stating the algebraic formula for this process is not sufficient for this mark to be awarded. This formula need not be fully correct, however it must be clear the candidate is attempting to select the right values for each part of the formula (e.g. condone slips in calculating $\sum x^2$ etc.).

- MC3: Mean, median and mode (where appropriate) calculated correctly:

These values must be stated correctly in the students' project (word file). These must be stated correct to three significant figures or better.

Note that seeing the correct values stated in the project is sufficient for this mark to be awarded.

- MC4: Attempt to find standard deviation and IQR for both data sets:

This attempt should be evidenced from the calculation shown in the students' project (word file). Minimum amount needed for the mark to be awarded is the formula for the standard deviation and IQR with values substituted in. For the IQR, calculations for Q_1, Q_2 and Q_3 should be seen. Stating the algebraic formula for this process is not sufficient for this mark to be awarded. This formula need not be fully correct, however it must be clear the candidate is attempting to select the right values for each part of the formula (e.g. condone slips in calculating $\sum x^2$ etc.). However, it must be clear that the candidate understands the difference between $\sum x^2$ and $(\sum x)^2$ in the standard deviation formula. Also, condone use of incorrect values found in for the mean and median being used in this formula.

- MC5: Standard deviation and IQR calculated correctly:

These values must be stated correctly in the students' project (word file). These must be stated correct to three significant figures or better. Allow for follow through here: if the candidate has applied the formula correctly using *their* incorrect values for the mean and the median, this mark should still be awarded.

Note seeing the correct values stated in the project is sufficient for this mark to be awarded.

- MC6: Outliers searched for and identified (if any):

This attempt should be evidenced from the calculation shown in the students' project (word file). A suitable test should be used for outliers using measures of location, or measures of spread etc. The most likely test to be used is: "Outliers are $> Q_3 + k(Q_3 - Q_1)$ and $< Q_1 - k(Q_3 - Q_1)$, where $1 < k \leq 2$", however alternative methods are accepted if correct. Minimum amount needed for the mark to be awarded is the formula used with values substituted in, so in the formula listed in the previous sentence "*their*" Q_3 and Q_1 should be seen. Stating the algebraic formula for this process is not sufficient for this mark to be awarded. A conclusion must be reached about whether the outliers exist which is consistent with the formula they have used for this mark.

- MC7: Attempt to find Sxx, Syy, Sxy:

This attempt should be evidenced from the calculation shown in the students' project (word file). Minimum amount needed for the mark to be awarded is the formulae for the Sxx, Syy and Sxy with values substituted in. All three formulae must be used. Stating the algebraic formula for this process is not sufficient for this mark to be awarded. This formula need not be fully correct, however it must be clear the candidate is attempting to select the right values for each part of the formula (e.g. condone slips in calculating $\sum x^2$ etc.).

- MC8: Sxx, Syy and Sxy calculated correctly:

These values must be stated correctly in the students' project (word file). These must be stated correct to three significant figures or better. Allow for follow through here: if the candidate has applied the formula correctly using *their* incorrect values for $\sum x^2$, $(\sum x)^2$ etc., this mark should still be awarded.

Note that the correct values stated in the project is sufficient for this mark to be awarded.

- MC9: PMCC calculated correctly:

This value must be stated correctly in the students' project (word file). Both the formula with values substituted in and the final correct value must be seen for this mark to be awarded. The mark can be awarded if the value of the PMCC is correct for the candidate's values of Sxx, Syy and Sxy, however they must reach a valid value of r; if the calculated value is not $-1 \leq r \leq 1$, then this mark cannot be given. It must also be clear the candidate is attempting to select the right values for each part of the formula (e.g. condone slips in calculating Sxx etc.).

Note that seeing the correct values stated in the project is sufficient for this mark to be awarded.

- MC10: Correct attempt made to find the equation of the regression line:

This attempt should be evidenced from the calculation shown in the students' project (word file). Minimum amount needed for the mark to be awarded is the formulae for *a* and *b* seen with values substituted in. Both formulae must be used. Stating the algebraic formula for this process is not sufficient for this mark to be awarded. This formula need not be fully correct, however it must be clear the candidate is attempting to select the right values for each part of the formula (e.g. condone slips in calculating Sxx etc.).

- MC11: Values of a and b in regression equation calculated correctly with correct working shown:

These values must be stated correctly in the students' project (word file). These must be stated correct to three significant figures or better. Allow for follow through here: if the candidate has applied the formula correctly using *their* incorrect values for Sxx etc., this mark should still be awarded.

Note that seeing the correct values stated in the project is sufficient for this mark to be awarded.

- MC12: Correct scatter graph (appropriate scale and labelled axes) with regression line plotted accurately on scatter graph:

Scatter graph must have an appropriate scale and both axes should be labelled. Units should be stated if the data has been coded in any way (e.g. if one of the variables is profit in thousands of pounds, the axes label should make it clear that the data is coded as such). Data points should not be connected: it must evidently be a scatter graph and not a line graph (or similar). The regression line should be seen drawn from the lowest x value to the highest x value as a minimum. The axes do not have to start from 0 if this affects the distribution of the data on the graph; an appropriate scale should be used and there should be a clear mark on the axis if it does not start from 0.

DATA INTERPRETATION (DI):

For all DI marks, where students are required to offer a reason, the responses do not have to be fully complete and correct. However, they should be sensible. The mark should be withheld if the reason given is clearly from fallacious reasoning.

NB: All of the marks below are dependent on MC1 being awarded.

- DI1: Correct comment on scatter graph (must address outliers, if any):

The analysis of the scatter graph should include a discussion of any outliers. These should be those identified in the MC6 calculation. Students should offer a reason as to why these data values are outliers: these do not have to be fully correct or rigorously justified but should be reasonable. Any obvious patterns in the data should also be discussed. For example, if the data displays a reasonably linear trend, but there is clearly another shape to the data as well, this should be mentioned (e.g. if the data looks like two piecewise linear functions, but they still have a similar enough gradient that $|PMCC| \geq 0.6$, the reasons why should be discussed. Similarly, if the data displays a strong correlation with little deviation from the regression line, a reason why this may be the case should also be offered.

- DI2: Comparison of data as univariate:

This comparison should talk about the averages and the spread of the data. They should compare the means and medians, and the standard deviation and IQR of the two data sets as a minimum. At least one measure of spread and one measure of location must be discussed for this mark to be awarded. They must give an interpretation of these values in context (e.g. the IQR of country A being higher than country B means that…). They should also give a reason as to why this may be the case. Students do not need to mention the skew of the data for this mark.

- DI3: Correct comment on PMCC (must refer to value), including idea that correlation does not imply causation:

The student needs to interpret the value of the PMCC rather than describing it. Note that their interpretation must be consistent with *their* value of the PMCC calculated earlier. They need to make it clear that a strong correlation means that as viable x increases, variable y increases/decreases. They also need to make it clear that they understand that one of these variables is not necessarily causing the other to change. The comment made should be specific to their data sets, rather than an abstract point about the relation between correlation and causation (e.g. despite there being a strong correlation, the population of country A increasing does not <u>cause</u> an increase in the population of country B).. Both a correct interpretation and a correlation/causation comment are needed for this mark to be awarded.

- DI4: Correct comment on the gradient of the regression line (must refer to the value):

Must be a fully correct interpretation. The student needs to make clear that for every one unit that variable x increases, variable y increases by b, where b is the gradient of the regression line. Note that their interpretation must be consistent with *their* value of b calculated earlier.

CONCLUSION (CN):

NB: CN1 and CN2 are dependent on SC1 and MC1 being awarded.

- CN1: Some correct ideas drawn from working towards answering the title question:

The student should refer back to their introduction and explain how their analysis of the data as bivariate helps answer this. As a minimum, they must refer to one of the gradient of regression line or the value of the PMCC for this mark. As a minimum for this mark, a broad attempt to conclude their research and a paragraph of writing should be seen.

- CN2: Fully correct conclusion including comments on intricacies of the data. All ideas are justified from prior work:

The student should refer to all the previous work they have completed to help answer the title question. This includes measures of location and spread of the univariate data, as well as the PMCC and regression gradient. In addition to this, they need to be clear that any conclusions they draw may not be reliable as correlation may not imply causation. Finally, there should be a prediction made based on their calculated regression line. However, students will need to make it clear that all the extrapolated predictions they make are unreliable.

STRUCTURE AND COMMUNICATION (SC):

- SC1: Attempt at logical structure including brief introduction:

The structure should be sensible, with a reasonable flow going from introduction (including the title "question"), to working with the data as univariate, to working with the data as bivariate, before finally concluding their ideas at the end. These sections do not need to be seen exactly, but there should be a clear progression of ideas. As a minimum, the introduction should include what the student hopes to achieve and where they intend to obtain the data from. Markers will be positive when awarding this mark.

- SC2: Full logical structure throughout. All ideas are appropriate and communicated clearly. Complete correct notation throughout.

All the above sections need to be seen distinctly. Notation and symbols need to be consistent and accurate. Subscripts and superscripts used where appropriate. Symbols used instead of words where possible and where appropriate. One slip is allowed, but there should be no consistent errors.

APPENDIX B. CONTROLLED ASSESSMENT DETAILS

The Controlled Assessment is worth 10% of the qualification.

Students will sit this assessment in a single two and a half hour period. They will be permitted to bring in any notes they have written, however they will not be able to use any printed notes or materials provided by their tutor. They will be expected to write their work on lined paper, which will be their final marked work, however there will be paper provided for their rough work and calculations. Graph paper will also be provided for students to construct their histograms. The assessment will be framed with the idea that students are working towards answering a question. For example, the students could be given two data sets about the hours of sunshine per day in a particular location over two time periods, before being asked to compare and contrast to decide when it is best to visit there. They may then be be told that there is one piece of data missing, or one piece of data in a set is erroneous and should be removed, before being asked how this would affect their calculations.

Unlike the Coursework, students will be provided with a data set to analyse at the beginning of the Controlled Assessment. They will be expected to calculate three measures of location, two measures of spread and the skew of the two data sets. They will be expected to compare and contrast these values for the two data sets and interpret their findings.

The marks will be awarded according to these criteria:

- 60% (12 marks) Mathematical Calculations
- 25% (5 marks) Data Interpretation
- 10% (2 marks) Conclusion
- 5% (1 mark) Structure and Communication (including Correct Terminology and Notation)

Students will be provided with the marking criteria prior to the start of the controlled assessment. The data set and context will be provided on the day of the assessment. Assessors will be provided with a full marking scheme with the correct answers and notes once this assessment is complete.

UFP MATHEMATICS CONTROLLED ASSESSMENT MARKING SHEET:

Name: ______________________________ Date: _________________

Mathematical Calculations (MC):	✓	×
MC1: Correctly states the modal class of the two data sets		
MC2: Attempts to find the mean of two data sets using a valid method		
MC3: Correctly finds the mean of the two data sets		
MC4: Attempts to find the median of the two data sets using a valid method		
MC5: Correctly finds the median of the two data sets		
MC6: Attempts to find the IQR of both data sets using a valid method		
MC7: Correctly finds the IQR of both data sets		
MC8: Attempts to find the standard deviation for both data sets using a valid method		
MC9: Correctly finds the deviation and variance for both data sets		
MC10: Correctly finds the frequency density for the specified class intervals		
MC11: Chooses an appropriate scale to draw both histograms		
MC12: Correctly draws both histograms		
TOTAL:	**/12**	

Data Interpretation (DI):	✓	×
DI1: Evalues the skew of the two data sets and chooses appropriate measures of location and spread to compare based on the skew of the data		
DI2: Correctly compares two measures of location		
DI3: Correctly compares two measures of spread		
DI4: Correctly compares the skew of the data		
DI5: Correctly explains how adding in an additional value/removing an existing value would affect all calculated values		
TOTAL:	**/5**	

Conclusion (CN):	✓	×
CN1: Some correct ideas drawn from working towards answering the question.		
CN2: Fully justified conclusion, referencing all prior working correctly.		
TOTAL:	**/2**	

Structure and Communication (SC):	✓	×
SC1: Full logical structure throughout. All ideas are appropriate and communicated clearly.* Complete correct notation throughout.		
TOTAL:	**/1**	

TOTAL		**/20**

* Note: Poor English should not be punished, but key words should be used and ideas should be clear.

MARKING CRITERIA CLARIFICATION:

*Note that **all** marks should be considered independent unless otherwise stated.*

Mathematical Calculations (MC):

- MC1: Correctly states the modal class of the two data sets:

This mark is evidenced from the student's written work.

- MC2: Attempts to find the mean of two data sets using a valid method:

The calculation for the mean should be seen for both sets of data. The numerator must be of the form $\sum fx$ and the denominator must be of the form $\sum f$. Minimum amount needed for the mark to be awarded is the formula for the mean with values substituted in. Stating the algebraic formula for this process is not sufficient for this mark to be awarded. This formula need not be fully correct, however it must be clear the candidate is attempting to select the right values for each part of the formula (e.g. condone slips in calculating $\sum fx$ etc.).

- MC3: Correctly finds the mean of the two data sets:

This mark is evidenced from the student's written work.

- MC4: Attempts to find the median of the two data sets using a valid method:

The calculation for the median must be seen for both data sets. There must be a clear attempt to use linear interpolation, where the cumulative frequencies and the modified (if appropriate) class boundaries are used. The formula need not be fully correct, however it must be clear the candidate is attempting to select the right values for each part of the formula (e.g. condone slips in calculating cumulative frequencies etc.).

- MC5: Correctly finds the median of the two data sets:

This mark is evidenced from the student's written work.

- MC6: Attempts to find the IQR of both data sets using a valid method:

The calculation for the IQR must be seen for both data sets. Like for the median, there must be a clear attempt to use linear interpolation for Q_1 and Q_3, where the cumulative frequencies and the modified (if appropriate) class boundaries are used. The Q_3 - Q_1 step must also be seen for both data sets.

- MC7: Correctly finds the IQR of both data sets:

This mark is evidenced from the student's written work.

- MC8: Attempts to find the standard deviation for both data sets using a valid method:

The calculation for the standard deviation must be seen for both data sets. Values of $\sum fx$ and $\sum f$ may be carried forward from their previous work to find the mean. There must be evidence that the student is attempting to find $\sum fx^2$ correctly and the values are placed into the formula correctly.

- MC9: Correctly finds the deviation and variance for both data sets:

This mark is evidenced from the student's written work.

- MC10: Correctly finds the frequency density for the specified class intervals:

This mark is evidenced from the student's written work.

- MC11: Chooses an appropriate scale to draw both histograms:

"Appropriate" in this context means that the histogram should:

  - Must not be too small such that it is hard to read or values are unnecessarily clustered.
  - The histogram must fit on the page with no missing information.

Note that the histogram must be drawn correctly from the calculated frequency densities.

- MC12: Correctly draws both histograms:

This mark is evidenced from the student's written work. In addition to the histogram being drawn correctly, all axes must be clearly labelled.

Note that much like for MC11, the mark can be awarded if the histogram is fully correct for the calculated frequency densities.

DATA INTERPRETATION (DI):

- DI1: Evaluates the skew of the two data sets and chooses appropriate measures of location and spread to compare based on the skew of the data:

Skew can be calculated by comparing the mean, median and mode of the data, or by comparing the differences between the quartiles to calculate skew. The minimum required is a mathematical statement (e.g. an inequality) and a written conclusion about the nature of the skew for this mark to be awarded.

For D12, DI3 and DI4: if the data selected is skewed, the students must use the median and IQR when comparing data. If the data is (close to) symmetrical, then the mean and standard deviation must be used. Where the value of the skew is ambiguous, then either calculation can be used, provided this is justified from the candidate's work.

- DI2: Correctly compares two measures of location:

Students should compare the appropriate measures of location for the two data sets and write a correct statement interpreting these values.

- DI3: Correctly compares two measures of spread:

Students should compare the appropriate measures of spread for the two data sets and write a correct statement interpreting these values.

- DI4: Correctly compares the skew of the data:

Students should compare the skew of the two data sets and write a correct statement interpreting these values.

- DI5: Correctly explains how adding in an additional value/removing an existing value would affect all calculated values:

Students are expected to be able to explain how an additional piece of data/removing an existing value would affect the mean, median and standard deviation for one of the data sets, without performing any additional calculations. Candidates are not penalised for recalculating these values, however this is not a requirement. Should they recalculate the values incorrectly, the conclusion they reach will not be treated as correct.

CONCLUSION (CN):

- CN1: Some correct ideas drawn from working towards answering the question:

There is evidence that the student is using the values they have calculated to compare the two data sets. For this mark to be awarded, at least one of DI2, DI3 and DI4 should have been awarded.

- CN2: Fully justified conclusion, referencing all prior working correctly.

The student should use all the values they have calculated so far to compare the two data sets. For this mark to be awarded, all DI2, DI3 and DI4 must have been awarded.

STRUCTURE AND COMMUNICATION (SC):

- SC1: Full logical structure throughout. All ideas are appropriate and communicated clearly. Complete correct notation throughout.

The written work should be presented clearly and ideas should flow in a logical manner. An introduction should be included; examples of this will be given in the marking scheme. Notation and symbols need to be consistent and accurate. Subscripts and superscripts used where appropriate. Symbols used instead of words where possible and where appropriate. One slip is allowed, but there should be no consistent errors.

APPENDIX C: TEACHING GUIDANCE AND CONTENT DETAILS

Algebra

Understand and use the laws of indices for all rational exponents:
Students should be able to simplify simple indical expressions and solve simple indical equations using standard rules:

$a^x \times a^y \equiv a^{x+y}$
$a^x \div a^y \equiv a^{x-y}$
$(a^x)^y \equiv a^{xy}$

Use and manipulate surds, including rationalising the denominator:
Students should be able to show how to simplify a surd by using $\sqrt{ab} = \sqrt{a}\sqrt{b}$ or $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$. They should also be able to rationalise the denominator for expressions of the form $\frac{a}{a\sqrt{b} \pm c\sqrt{d}}$.

Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the conditions for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown:
Students should be able to sketch a quadratic graph of the form $y = ax^2 + bx + c$ for all values of a,b and c. They should understand the relationship between the discriminant and the number of intersections between the curve and the x-axis. For quadratic curves that do not cross the x-axis, they should be able to use the completing the square to identify the vertex and display this point on their sketches. Students should be able to solve quadratic equations using a variety of techniques. They should be able to apply all of these ideas to problems in context, such as simple projectile models. Students should also know that the discriminant can be used to find when equations have no, repeated or distinct solutions and use this in coordinate geometry problems, such as finding when a line is a tangent to a circle.

Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation:
Students should be able to solve a system of two simultaneous equations to find two unknown variables. They should understand that graphically, this represents the intersection point of two curves.

Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions:
Students should be able to solve quadratic inequalities and understand the link between the solutions of the inequality and the graph of the curve. They should be able to solve any inequality of the form $f(x) \geq 0$ or $f(x) \leq 0$ if they are given a graph of the function.

Represent linear and quadratic inequalities such as $y > x + 1$ and $y > ax^2 + bx + c$ graphically.
Students should be able to identify regions of the plane defined by multiple lines and quadratic curves.

Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem. Simplify rational expressions including by factorising and cancelling, and algebraic division (by linear expressions only):

Students should be able to use the factor theorem to identify factors of a polynomial expression. They should be able to use polynomial long division to simplify expressions of the form $\frac{f(x)}{(x-a)}$ and hence to factorise the polynomial $f(x)$. In addition, they should be able to simplify algebraic fractions by factorising the numerator and denominator and cancelling like terms.

Understand and use graphs of functions; sketch curves defined by simple equations including polynomials, the modulus of a linear function, $y = \frac{a}{x}$ and $y = \frac{a}{x^2}$ (including their vertical and horizontal asymptotes); interpret algebraic solution of equations graphically; use intersection points of graphs to solve equations. Understand and use proportional relationships and their graphs:

Students should be able to sketch polynomials up to and including degree 4. They should be able to sketch cubic and quartic curves expressed in factorised form. Sketches should include any and all intersections with the axes but do not need to include extrema unless specifically requested. Students are expected to be able to sketch hyerpbolae of the form $f(x) = \frac{k}{(x-a)} + b$, for any constants a,b and k. These sketches should show all asymptotes and and their equations. Students may be asked to use linear models to solve "real-life" problems and explain/interpret their work where appropriate. Students should also be able to sketch graphs of the form $y = a|(x-b)| + c$.

Understand and use composite functions; inverse functions and their graphs:

Students should be able to know the definition of a function (both many-to-one and one-to-one). They should be able to find the range of a function for a given domain. Given two functions, $f(x)$ and $g(x)$, they should be able to find $fg(x)$ and $gf(x)$. They should also be able to find $f^{-1}(x)$ (if f is one-to-one) and know the relationship between the domains and ranges of $f(x)$ and $f^{-1}(x)$. They should also know how the graphs of $f(x)$ and $f^{-1}(x)$ are related.

Understand the effect of simple transformations on the graph of $y = f(x)$, including sketching associated graphs: $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.

Students could be provided with a sketch of $y = f(x)$ and be asked to transform it, ensuring that they show how all points and asymptotes are affected by this transformation. It is also expected that they are able to describe transformations in words.

Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear):

The denominators will be in one of these forms:

- $(x-a)(x-b)$
- $(x-a)(x-b)(x-c)$
- $(x-a)(x-b)^2$

If the degree of the numerator is higher than the degree of the denominator, students are expected to use polynomial long division first.

Coordinate Geometry

Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; gradient conditions for two straight lines to be parallel or perpendicular. Be able to use straight line models in a variety of contexts:

Students are expected to be able to use standard coordinate geometry techniques to find:

- Equations of straight lines with any gradient and y-intercept, including vertical and horizontal lines.
- Intersection points through simultaneous equations
- Lengths, mid-points and grdients of line segments
- Equations of straight lines given a point and gradient or two points
- Lines parallel or perpendicular to an original point.

They are expected to be able to choose an appropriate procedure to find other properties of objects in the x-y plane, such as the area of triangles formed.

Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$; completing the square to find the centre and radius of a circle; use of the following properties:

- the angle in a semicircle is a right angle
- the perpendicular from the centre to a chord bisects the chord
- the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point:

The equation of the circle may be given in its expanded form and students may need to complete the square to find the centre and radius. Students may be asked to solve a wide variey of coordinate geometry problems using the circle theorems listed above. These problems can often be solved either algebraically or through use of a diagram. It is expected that they are conversant with the properties of straight lines when solving exam questions on circles.

Sequences and series

Understand and use the binomial expansion of $(a+b)^n$ for positive integer *n*; the notations *n*! and *n*C*r*; link to binomial probabilities.

Students need to be able to find values of nCr and explain their meaning in the context of choosing r objects from a set of n. They should be able to expand $(a+b)^n$ for natural powers of n where a and b are either numbers of multiples of x. They also need to be able to solve problems in context, such as the probability of a certain event happening r times out of n total. Students are expected to be able to find these probabilities through use of a graphical calculator.

Understand and use sigma notation for sums of series:

$\sum_{r=1}^{n} f(r)$ will be used for both arithmetic and geometric sequences.

Understand and work with arithmetic sequences and series, including the formulae for *n*th term and the sum to *n* terms:

Students will need to use the formulae $U_n = a + (n-1)d$ and $U_n = ar^{n-1}$ to identify the nth terms of arithmetic and geometric sequences respectively.

Understand and work with geometric sequences and series including the formulae for the *n*th term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $|r| < 1$; modulus notation

Students will need to use the formulae $S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(a+L)$ and $S_n = \frac{a(1-r^n)}{(1-r)}$ for arithmetic and geometric sequences respectively. They will also need to understand the conditions for the infinite sum of a geometric series to be convergent and to be able to use the formula $S_n = \frac{a}{(1-r)}$.

Use sequences and series in modelling:

Students may be asked to solve sequences questions that model real life situations.

Trigonometry

Understand and use the definitions of sine, cosine and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form $\frac{1}{2}absinC$:

Students are expected to be able to use calculators to find missing sides and angles in right andgled triangles using SOHCAHTOA formulae. For non-right angled triangles, they are expected to be able to use the cosine and sine rules to identify missing sides and angles, including the ambiguous case for angles with the sine rule. This also includes problems in context, such as finding bearings. Students are also expected to find the area of scalene triangles.

Work with radian measure, including use for arc length and area of sector:

Students are expected to be able to convert between the degree and radian form of an angle. They will need to use $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ to find arc lengths and areas of sectors respectively, and be able to find the area of a segment by considering it as a sector minus a triangle.

Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity:

Students should be able to know the graphs of $\sin(x)$, $\cos(x)$ and $\tan(x)$ and be aware of key points, expressed both as degrees and radians.

Know and use exact values of sin and cos for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples thereof, and exact values of tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \pi$ and multiples thereof:

Students will be able to use calculators to find these values.

Understand and use $tan\theta = \frac{sin\theta}{cos\theta}$:

Students may be asked to simplify trigonometric expressions or equations using this identity.

Understand and use $sin^2\theta + cos^2\theta = 1$:

Students may be asked to simplify trigonometric expressions or equations using this identity.

Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle:

Students will be expected to be able to solve trigonometric equations in terms of sin, cos or tan for any finite domain. This includes both linear and quadratic equations, where the use of trigonmetric identities may be necessary. Students may also be given trigonometric equations involving linear transformations of x, such as $\sin(2x) = \frac{1}{2}$. These equations can be in either degrees or radians.

Construct proofs involving trigonometric functions and identities:

These will be given as "show" questions, where students are expected to show that the two sides are equivalent.

Use trigonometric functions to solve problems in context:

Students may be expected to apply these ideas to problems in context.

Exponentials and Logarithms

Know and use the function a^x and its graph, where a is positive:
Students should be able to sketch the graph of $y = a^x$ for all $a > 0$. Sketches should show the intersection with the vertical axis and make it clear that the x-axis is an asymptote.

Know and use the function e^x and its graph.
Students should be familiar with the number e and the fact that the graph of $y = e^x$ is its own gradient function.

Know that the gradient of is e^{kx} equal to ke^{kx} and hence understand why the exponential model is suitable in many applications.
This gradient function will also need to be shown using the chain rule for differentiation.

Know and use the definition of $\log_a x$ as the inverse of a^x, where $a > 0$ and $x > 0$.
Students will be expected to convert between exponential and logarithmic form, i.e. $a^x = n \Leftrightarrow n = \log_a x$ for a >0 and x >0.

Know and use the function and $\ln x$ its graph.
Students must show the shape of the graph, the intersection with the x-axis and make it clear that the y-axis is an asymptote on their sketch.

Know and use as the inverse function of e^x.
They should understand that e^x and $\ln x$ are a special case of $a^x = n \Leftrightarrow n = \log_a x$.

Understand and use the laws of logarithms: $\log_a x + \log_a y = \log_a xy$; $\log_a x - \log_a y = \log_a \frac{x}{y}$; $k\log_a x = \log_a x^k$ including, for example, $k = -1$ and $k = \frac{-1}{2}$.
Solve equations of the form $a^x = b$.
Students may be asked to solve logarithmic equations where all log functions have a consistent base. This may involve simplifying terms using the rules of logarithms. They also may be asked to solve exponential equations of the form $a^x = b$ by using logarithms.

Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y:
Students may be asked to identify constants a and n, or b and k, by converting a data set using logarithms. They may then be asked to draw a trend line through this data, and using the gradient and y-intercept of this line, find the original values a and n, or b and k.

Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models.
Students may be asked to identify the constants a and n, or b and k in $y = ax^n$ and $y = kb^x$, when working with mathematical models.

Differentiation

Understand and use the derivative of as the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second derivatives. Understand and use the second derivative as the rate of change of gradient:
Students should understand that $f(x)$, $f'(x)$ and $f''(x)$ all represent different features of a curve.

Differentiate x^n, for rational values of *n*, and related constant multiples, sums and differences.
Students should be able to differentiate ax^n for all values of a and n. Some expressions may need simplifying using the rules of indices before they are differentiated.

Differentiate e^x and a^{kx}, $sinkx$ and $coskx$ related sums, differences and constant multiples. Understand and use the derivative of *ln x:*
Students should be able to differentiate these and understand how to apply the chain rule to these functions.

Apply differentiation to find gradients, tangents and normals, maxima and minima and stationary points, points of inflection. Construct simple differential equations in pure mathematics and in context:
Students may be asked to use tangents and normal in coordinate geometry problems; this does not include differentiating the circle. Students may be asked to solve "real-life" problems in context by identifying optimal solutions (maxima and minima) in mathematical models. Students will be expected to justify that a stationary point is a maximum or minimum either by using the second derivative or by examining the gradient either side of the point.

Identify where functions are increasing or decreasing:
This may be asked in theoretical and "real-life"contexts.

Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions:
Students may be asked to differentiate any combination of $ax^n, e^x, sinx, cosx$ and $\ln x$. Questions may include a combination or the product and chain rules, or the quotient and chain rules. Where more than one method is viable, all correct methods will be given equal credit. Questions involving the chain, product and quotient rules may also ask students to find equations of tangents and normals, or to find stationary points. For the chain rule, students are expected to be able to use $\frac{dx}{dy}$ to find an expression for $\frac{dy}{dx}$.

Integration

Know and use the Fundamental Theorem of Calculus:
Students are expected to use $\int_a^b f(x)\,dx = F(b) - F(a)$.

Integrate x^n (excluding *n* = -1), e^{kx}, $\frac{1}{x}$, $sinkx$, $coskx$ and related sums, differences and constant multiples. Carry out simple cases of integration by substitution and integration by parts. Integrate using partial fractions that are linear in the denominator
Students may be asked to use a variety of techniques, such as substitution, parts or partial fractions, to integrate expressions. Questions will inform the student of which technique is necessary when solving each integral:

- When partial fractions are required, simplifying the expression will be an earlier part of the question.
- When integration by substitution is required, the expression for either u or u^2 will be given.
- When parts are required, the question will state this but will not state which expression should be u and which should be $\frac{dv}{dx}$.

Evaluate definite integrals; use a definite integral to find the area under a curve and the area between two curves:
When finding the area between two curves, students will be able to use $A = \int_a^b (y_2 - y_1)\,dx$, where y_2 and y_1 are the two curves and a and b are the intersection points. Geometric reasoning, such as combining an integral with the area of a square, triangle, trapezium etc., will also be acceptable for certain questions. Students may have to find the area of curves that straddle the x-axis; it is expected that the integral is split up into separate parts for these questions.

Data Presentation and Interpretation (Coursework)

Construct and interpret diagrams for discrete single-variable data:
Students will be expected to arrange data in a frequency table and be able to analyse data when it is presented in this way. They will also be expected to analyse and construct scatter diagrams.

Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams:
Students will be expected to interpret the gradient of a regression line by stating it's "real-life" meaning in terms of the vairables x and y.

Understand informal interpretation of correlation. Understand that correlation does not imply causation:
Students have to understand that correlation implies that as one variable increases, the other increases or decreases (depending on the type of correlation). However, they need to justify whether or not one variable is *causing* the other variable to change, as in most cases this is not true.

Be able to calculate measures of central tendency and variation, including standard deviation, for discrete data:
Students should be able to find the mean, median and mode for a set of data. They should also be able to find the range, inter-quartile range, variance and standard deviation.

The mode should be the value with the greatest frequency. The median and quartiles should be found by looking at the values in positions $\frac{n}{4}, \frac{n}{2}$ and $\frac{3n}{4}$. The mean should be found by $\bar{x} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$. The standard deviation should be found by $\sigma = \sqrt{\frac{\sum x^2}{n} - (\frac{\sum x}{n})^2} = \sqrt{\frac{\sum fx^2}{\sum f} - (\frac{\sum fx}{\sum f})^2}$ and the variance is the square of this.

Interpret measures of central tendency and variation, extending to standard deviation, for discrete data:
Students should be able to explain what each of these values represents in the context of the question and be able to use these values to compare two data sets. For example, if one data set has a greater mean than another, then the student should be able to write a statement concluding that set A has a higher average than set B. If one data set has a greater standard deviation, then the student should be able to conclude that one set has greater diversity in its data than the other.

Data Presentation and Interpretation (Controlled Assessment)

Construct and interpret diagrams for single-variable continuous data, including understanding that area in a histogram represents frequency
Students should be able to construct a histogram given data in a frequency table by first calculating the frequency densities. When interpreting, they should be able to calculate frequencies from areas of the bars in a histogram and vice versa. Students should also be able to explain that a histogram is used to represent continuous data and understand that the bars in the histogram represent frequencies.

Be able to calculate measures of central tendency and variation, extending to standard deviation for continuous data:
Students should be able to find the mean, median and mode for a set of data. They should also be able to find the inter-quartile range, variance and standard deviation.

The mode should be the value with the greatest frequency. The median and quartiles should be found by using linear interpolation to identify the values in positions $\frac{n}{4}, \frac{n}{2}$ and $\frac{3n}{4}$. The mean should be found by $\bar{x} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$, where the x value is the mid-point of the class. The standard deviation should be found by $\sigma = \sqrt{\frac{\sum x^2}{n} - (\frac{\sum x}{n})^2} = \sqrt{\frac{\sum fx^2}{\sum f} - (\frac{\sum fx}{\sum f})^2}$ and the variance is the square of this.

Interpret measures of central tendency and variation, extending to standard deviation for continuous data:
Students should be able to explain what each of these values represents in the context of the question and be able to use these values to compare two data sets. For example, if one data set has a greater mean than another, then the student should be able to write a statement concluding that set A has a higher average than set B. If one data set has a greater standard deviation, then the student should be able to conclude that one set has greater diversity in its data than the other.

Select or critique data presentation techniques and compare data in the context of a statistical problem:
When comparing data, students must discuss a measure or location, a measure of spread and the skew of the data. If the data is skewed, students should know to use the median and quartiles when comparing. If the data is near to symmetric, the should know to use the mean and standard deviation.

Be able to calculate and compare the skew of two data sets:
Students should be able to calculate the skew by one of:

- Comparing the mean, median and mode
- Comparing the difference between Q_3 and Q_2, and the difference between Q_2 and Q_1.

APPENDIX D. SYMBOLOGY

1. Set Notation

1.1 $\in$	is an element of
1.2 $\notin$	is not an element of
1.3 $\subseteq$	is a subset of
1.4 $\subset$	is a proper subset of
1.5 $\{x_1,x_2,\ldots,x_k\}$	the set with elements $x_1,x_2,\ldots,x_k$
1.6 $\{x : \ldots\}$	the set of all x such that …
1.7 n(A)	the number of elements in set A
1.8 $\emptyset$	the empty set
1.9 ε	the universal set
1.10 A′	the complement of the set A
1.11 $\mathbb{N}$	the set of natural numbers, {1, 2, 3, …}
1.12 $\mathbb{Z}$	the set of integers, {0, ±1, ±2, ±3, …}
1.13 $\mathbb{Z}^+$	the set of positive integers, {1, 2, 3, …}
1.14 $\mathbb{Z}_0^+$	the set of non-negative integers, {0, 1, 2, 3, …}
1.15 $\mathbb{R}$	the set of real numbers
1.16 $\mathbb{Q}$	the set of rational numbers, $\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\}$
1.17 $\cup$	union
1.18 $\cap$	intersection
1.19 (x,y)	the ordered pair x , y
1.20 [a,b]	the closed interval $\{x \in \mathbb{R}: a \le x \le b\}$
1.21 [a,b)	the interval $\{x \in \mathbb{R}: a \le x < b\}$
1.22 (a,b]	the interval $\{x \in \mathbb{R}: a < x \le b\}$
1.23 (a,b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. Miscellaneous Symbols

2.1 $=$	is equal to
2.2 $\neq$	is not equal to
2.3 $\equiv$	is identical to or is congruent to
2.4 $\approx$	is approximately equal to
2.5 ∞	infinity
2.6 $\propto$	is proportional to
2.7 $\therefore$	therefore
2.8 $\because$	because
2.9 $<$	is less than
2.10 $\leqslant$	is less than or equal to, is not greater than
2.11 $>$	is greater than
2.12 $\geqslant$	is greater than or equal to, is not less than
2.13 $p \Rightarrow q$	p implies q (if p then q)
2.14 $p \Leftarrow q$	p is implied by q (if q then p)
2.15 $p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
2.16 a	first term for an arithmetic or geometric sequence
2.17 l	last term for an arithmetic sequence
2.18 d	common difference for an arithmetic sequence
2.19 r	common ratio for a geometric sequence
2.20 S_n	sum to n terms of a sequence
2.21 S_∞	sum to infinity of a sequence

3. Operations

3.1 a + b	a plus b
3.2 a − b	a minus b
3.3 a × b, ab, a·b	a multiplied by b
3.4 a ÷ b, $\frac{a}{b}$	a divided by b
3.5 $\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \cdots + a_n$
3.6 $\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
3.7 $\sqrt{a}$	the non-negative square root of a
3.8 |a|	the modulus of a
3.9 n!	n factorial: $n! = n \times (n-1) \times \ldots \times 2 \times 1, n \in \mathbb{N}$, $0! = 1$
3.10 $\binom{n}{r}$, ${}^{n}C_r$	the binomial coefficient $\frac{n!}{(n-r)!\,r!}$ For $n, r \in \mathbb{Z}_0^+$, r ⩽ n

4. Functions

4.1 $f(x)$	the value of the function f at x
4.2 $f: x \mapsto y$	the function f maps the element x to the element y
4.3 f^{-1}	the inverse function of the function f
4.4 gf	the composite function of f and g which is defined by gf(x) = g(f(x))
4.5 $\lim_{x \to a} f(x)$	the limit of f(x) as x tends to a
4.6 Δx, δx	an increment of x
4.7 $\frac{dy}{dx}$	the derivative of y with respect to x
4.8 $\frac{d^n y}{dx^n}$	the n th derivative of y with respect to x
4.9 $f'(x), f''(x), \ldots, f^{(n)}(x)$	the first, second, ..., n th derivatives of f $f(x)$ with respect to x
4.10 $\dot{x}, \ddot{x}$,…	the first, second, ... derivatives of x with respect to t
4.11 $\int y \ dx$	the indefinite integral of y with respect to x
4.12 $\int_a^b y \ dx$	the definite integral of y with respect to x between the limits x = a and x = b

5. Exponential and Logarithmic Functions

5.1 e	base of natural logarithms
5.2 e^x, exp x	exponential function of x
5.3 $log_a x$	logarithm to the base a of x
5.4 $\ln x$, $log_e x$	natural logarithm of x

6 Trigonometric Functions

6.1 sin, cos, tan	the trigonometric functions
6.2 arcsin, arccos, arctan	
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$	the inverse trigonometric functions
6.3 °	degrees
6.4 rad	radians

7 Probability and Statistics

7.1 P(A)	probability of the event A
7.2 A'	complement of the event A
7.3 X, Y, R etc.	random variables
7.4 B(n,p)	binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in a trial
7.5 q	q = 1 - p for binomial distribution

Pure Mathematics:

Quadratic Equations:

$ax^2 + bx + c = 0$ has roots $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of Indices:

$a^x \times a^y \equiv a^{x+y}$
$a^x \div a^y \equiv a^{x-y}$
$(a^x)^y \equiv a^{xy}$

Laws of Logarithms:

$a^x = n \Leftrightarrow n = log_a x$ for a >0 and x >0
$log_a x + log_a y \equiv log_a xy$
$log_a x - log_a y \equiv log_a \frac{x}{y}$
$k log_a x \equiv log_a (x^k)$

Coordinate Geometry:

A straight line graph, gradient m, passing through (x_1, y_1), has equation $y - y_1 = m(x - x_1)$.
Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$.

Sequences:

General term of an arithmetic progression: $U_n = a + (n-1)d$
General term of a geometric progression: $U_n = ar^{n-1}$

Trigonometry:

In the triangle ABC:
Sine rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$
Area: $A = \frac{1}{2} ab \sin C$

Mensuration:
Circumference and Area of circle, radius r and diameter d:
$C = 2\pi r = \pi d$ $\qquad$ $A = \pi r^2$

Pythagoras' Theorem: In any right-angled triangle where a, b and c are the lengths of the sides and c is the hypotenuse:
$a^2 + b^2 = c^2$
Area of a trapezium: $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation.
Volume of a prism = area of cross section × length
For a circle of radius r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A:
$s = r\theta \qquad A = \frac{1}{2}r^2\theta$

Calculus:
Differentiation:

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration:

Function	Derivative
x^n	$\frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin kx$	$\frac{-1}{k}\cos kx + c$
e^{kx}	$\frac{1}{k}e^{kx} + c$
$\frac{1}{x}$	$\ln\lvert x\rvert + c, x \neq 0$
$f'(x) + g'(x)$	$f(x) + g(x) + c$
$f'(g(x))g'(x)$	$f\big(g(x)\big) + c$

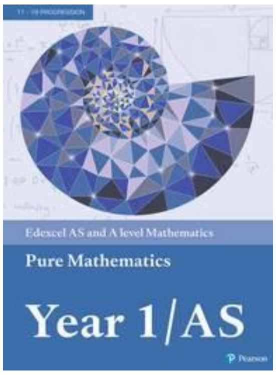
Area under a curve:

$$A = \int_a^b y\,dx\,, (y \geq 0)$$

Statistics:

The mean of a set of data: $\bar{x} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$

APPENDIX F. SUGGESTED EXTENSION READING



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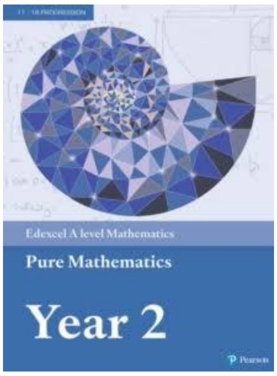

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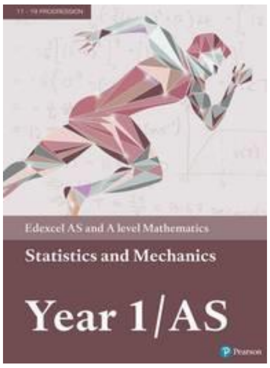

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APPENDIX G. FORMULA BOOK

Arithmetic Series:

$U_n = a + (n-1)d$

$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(a + l)$

Geometric Series:

$U_n = ar^{n-1}$

$S_n = \frac{a(1-r^n)}{(1-r)}$

$S_\infty = \frac{a}{(1-r)}$ for $|r| < 1$

Binomial Series:

$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \cdots + {}^nC_r a^{n-r}b^r + \cdots + b^n \quad (n \in \mathbb{N})$

where ${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \cdots$

where $|x| < 1, n \in \mathbb{R}$

Exponentials and Logarithms:

$log_b a = \frac{log_c a}{log_c b}$

$a^x = e^{xlna}$

Trigonometry:

Sine rule: $\frac{Sin\,A}{a} = \frac{Sin\,B}{b} = \frac{Sin\,C}{c}$

Cosine rule: $a^2 = b^2 + c^2 - 2cbcosA$

Area: $A = \frac{1}{2}absinC$

Calculus:

Quotient Rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$

Integration by parts: $\int u\frac{dv}{dx}dx = [uv] - \int v\frac{du}{dx}dx$

Binomial probabilities:

$P(X = r) = \binom{n}{r}\, p^r(1-p)^{n-r}$

Cumulative Binomial Probability Tables:

	c	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95
		p										
n = 1	0	0.950	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.050
	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 2	0	0.903	0.810	0.640	0.490	0.360	0.250	0.160	0.090	0.040	0.010	0.003
	1	0.998	0.990	0.960	0.910	0.840	0.750	0.640	0.510	0.360	0.190	0.098
	2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 3	0	0.857	0.729	0.512	0.343	0.216	0.125	0.064	0.027	0.008	0.001	0.000
	1	0.993	0.972	0.896	0.784	0.648	0.500	0.352	0.216	0.104	0.028	0.007
	2	1.000	0.999	0.992	0.973	0.936	0.875	0.784	0.657	0.488	0.271	0.143
	3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 4	0	0.815	0.656	0.410	0.240	0.130	0.063	0.026	0.008	0.002	0.000	0.000
	1	0.986	0.948	0.819	0.652	0.475	0.313	0.179	0.084	0.027	0.004	0.000
	2	1.000	0.996	0.973	0.916	0.821	0.688	0.525	0.348	0.181	0.052	0.014
	3	1.000	1.000	0.998	0.992	0.974	0.938	0.870	0.760	0.590	0.344	0.185
	4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

	c	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95
n = 5	0	0.774	0.590	0.328	0.168	0.078	0.031	0.010	0.002	0.000	0.000	0.000
	1	0.977	0.919	0.737	0.528	0.337	0.188	0.087	0.031	0.007	0.000	0.000
	2	0.999	0.991	0.942	0.837	0.683	0.500	0.317	0.163	0.058	0.009	0.001
	3	1.000	1.000	0.993	0.969	0.913	0.813	0.663	0.472	0.263	0.081	0.023
	4	1.000	1.000	1.000	0.998	0.990	0.969	0.922	0.832	0.672	0.410	0.226
	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 6	0	0.735	0.531	0.262	0.118	0.047	0.016	0.004	0.001	0.000	0.000	0.000
	1	0.967	0.886	0.655	0.420	0.233	0.109	0.041	0.011	0.002	0.000	0.000
	2	0.998	0.984	0.901	0.744	0.544	0.344	0.179	0.070	0.017	0.001	0.000
	3	1.000	0.999	0.983	0.930	0.821	0.656	0.456	0.256	0.099	0.016	0.002
	4	1.000	1.000	0.998	0.989	0.959	0.891	0.767	0.580	0.345	0.114	0.033
	5	1.000	1.000	1.000	0.999	0.996	0.984	0.953	0.882	0.738	0.469	0.265
	6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

		p										
	c	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95
n = 7	0	0.698	0.478	0.210	0.082	0.028	0.008	0.002	0.000	0.000	0.000	0.000
	1	0.956	0.850	0.577	0.329	0.159	0.063	0.019	0.004	0.000	0.000	0.000
	2	0.996	0.974	0.852	0.647	0.420	0.227	0.096	0.029	0.005	0.000	0.000
	3	1.000	0.997	0.967	0.874	0.710	0.500	0.290	0.126	0.033	0.003	0.000
	4	1.000	1.000	0.995	0.971	0.904	0.773	0.580	0.353	0.148	0.026	0.004
	5	1.000	1.000	1.000	0.996	0.981	0.938	0.841	0.671	0.423	0.150	0.044
	6	1.000	1.000	1.000	1.000	0.998	0.992	0.972	0.918	0.790	0.522	0.302
	7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 8	0	0.663	0.430	0.168	0.058	0.017	0.004	0.001	0.000	0.000	0.000	0.000
	1	0.943	0.813	0.503	0.255	0.106	0.035	0.009	0.001	0.000	0.000	0.000
	2	0.994	0.962	0.797	0.552	0.315	0.145	0.050	0.011	0.001	0.000	0.000
	3	1.000	0.995	0.944	0.806	0.594	0.363	0.174	0.058	0.010	0.000	0.000
	4	1.000	1.000	0.990	0.942	0.826	0.637	0.406	0.194	0.056	0.005	0.000
	5	1.000	1.000	0.999	0.989	0.950	0.855	0.685	0.448	0.203	0.038	0.006
	6	1.000	1.000	1.000	0.999	0.991	0.965	0.894	0.745	0.497	0.187	0.057
	7	1.000	1.000	1.000	1.000	0.999	0.996	0.983	0.942	0.832	0.570	0.337
	8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

		p										
	c	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95

n = 9	0	0.630	0.387	0.134	0.040	0.010	0.002	0.000	0.000	0.000	0.000	0.000
	1	0.929	0.775	0.436	0.196	0.071	0.020	0.004	0.000	0.000	0.000	0.000
	2	0.992	0.947	0.738	0.463	0.232	0.090	0.025	0.004	0.000	0.000	0.000
	3	0.999	0.992	0.914	0.730	0.483	0.254	0.099	0.025	0.003	0.000	0.000
	4	1.000	0.999	0.980	0.901	0.733	0.500	0.267	0.099	0.020	0.001	0.000
	5	1.000	1.000	0.997	0.975	0.901	0.746	0.517	0.270	0.086	0.008	0.001
	6	1.000	1.000	1.000	0.996	0.975	0.910	0.768	0.537	0.262	0.053	0.008
	7	1.000	1.000	1.000	1.000	0.996	0.980	0.929	0.804	0.564	0.225	0.071
	8	1.000	1.000	1.000	1.000	1.000	0.998	0.990	0.960	0.866	0.613	0.370
	9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

	c	p 0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95
n = 10	0	0.599	0.349	0.107	0.028	0.006	0.001	0.000	0.000	0.000	0.000	0.000
	1	0.914	0.736	0.376	0.149	0.046	0.011	0.002	0.000	0.000	0.000	0.000
	2	0.988	0.930	0.678	0.383	0.167	0.055	0.012	0.002	0.000	0.000	0.000
	3	0.999	0.987	0.879	0.650	0.382	0.172	0.055	0.011	0.001	0.000	0.000
	4	1.000	0.998	0.967	0.850	0.633	0.377	0.166	0.047	0.006	0.000	0.000
	5	1.000	1.000	0.994	0.953	0.834	0.623	0.367	0.150	0.033	0.002	0.000
	6	1.000	1.000	0.999	0.989	0.945	0.828	0.618	0.350	0.121	0.013	0.001
	7	1.000	1.000	1.000	0.998	0.988	0.945	0.833	0.617	0.322	0.070	0.012
	8	1.000	1.000	1.000	1.000	0.998	0.989	0.954	0.851	0.624	0.264	0.086
	9	1.000	1.000	1.000	1.000	1.000	0.999	0.994	0.972	0.893	0.651	0.401
	10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

	c	p										
		0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95
n = 12	0	0.540	0.282	0.069	0.014	0.002	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.882	0.659	0.275	0.085	0.020	0.003	0.000	0.000	0.000	0.000	0.000
	2	0.980	0.889	0.558	0.253	0.083	0.019	0.003	0.000	0.000	0.000	0.000
	3	0.998	0.974	0.795	0.493	0.225	0.073	0.015	0.002	0.000	0.000	0.000
	4	1.000	0.996	0.927	0.724	0.438	0.194	0.057	0.009	0.001	0.000	0.000
	5	1.000	0.999	0.981	0.882	0.665	0.387	0.158	0.039	0.004	0.000	0.000
	6	1.000	1.000	0.996	0.961	0.842	0.613	0.335	0.118	0.019	0.001	0.000
	7	1.000	1.000	0.999	0.991	0.943	0.806	0.562	0.276	0.073	0.004	0.000
	8	1.000	1.000	1.000	0.998	0.985	0.927	0.775	0.507	0.205	0.026	0.002
	9	1.000	1.000	1.000	1.000	0.997	0.981	0.917	0.747	0.442	0.111	0.020
	10	1.000	1.000	1.000	1.000	1.000	0.997	0.980	0.915	0.725	0.341	0.118
	11	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.986	0.931	0.718	0.460
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

	c	p										
		0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95
n = 15	0	0.463	0.206	0.035	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.829	0.549	0.167	0.035	0.005	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.964	0.816	0.398	0.127	0.027	0.004	0.000	0.000	0.000	0.000	0.000
	3	0.995	0.944	0.648	0.297	0.091	0.018	0.002	0.000	0.000	0.000	0.000
	4	0.999	0.987	0.836	0.515	0.217	0.059	0.009	0.001	0.000	0.000	0.000
	5	1.000	0.998	0.939	0.722	0.403	0.151	0.034	0.004	0.000	0.000	0.000
	6	1.000	1.000	0.982	0.869	0.610	0.304	0.095	0.015	0.001	0.000	0.000
	7	1.000	1.000	0.996	0.950	0.787	0.500	0.213	0.050	0.004	0.000	0.000
	8	1.000	1.000	0.999	0.985	0.905	0.696	0.390	0.131	0.018	0.000	0.000
	9	1.000	1.000	1.000	0.996	0.966	0.849	0.597	0.278	0.061	0.002	0.000
	10	1.000	1.000	1.000	0.999	0.991	0.941	0.783	0.485	0.164	0.013	0.001
	11	1.000	1.000	1.000	1.000	0.998	0.982	0.909	0.703	0.352	0.056	0.005
	12	1.000	1.000	1.000	1.000	1.000	0.996	0.973	0.873	0.602	0.184	0.036
	13	1.000	1.000	1.000	1.000	1.000	1.000	0.995	0.965	0.833	0.451	0.171
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.995	0.965	0.794	0.537
	15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000